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GENERALIZED CLASSES OF NEARLY OPEN SETS

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ABSTRACT

In this paper, we introduce and study the notion of generalized M-closed sets. Further, The notion of generalized M-open sets and some of its basic properties are introduced discussed. Moreover, we introduce the forms of generalized M-closed functions. We obtain some characterizations and properties are investigated.

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I. INTRODUCTION

EL-Magharabi and AL-Juhani, in 2011 [8] introduced and studied the notion of M-open sets. The class of g-closed sets was investigated by Aull in 1968 [2]. Umehara et.al [11] (resp. Dhana Balan [4]) introduced the concept of gp-closed (resp. ge-closed) sets. In this paper, we define and study the notion gM-closed sets and gM-open sets which is stronger than the concept of ge-closed and weaker than of gp-closed and M-closed sets. Also, some characterizations of these concepts are discussed. Moreover, we introduce and study new forms of generalized M-closed functions. We obtain properties of these new forms of generalized M-closed functions and preservation theorems.

Throughout this paper (X, τ) and (Y, σ) (simply, X and Y) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and X\A denote the closure of A, the interior of A and the complement of A respectively. A point $x \in X$ is called a δ-adherent point of A [14] if $A \cap int(cl(V)) \neq \emptyset$, for every open set V containing x. The set of all δ-adherent points of A is called the δ-closure of A and is denoted by $cl_\delta(A)$. A subset A of X is called δ-closed if $A= cl_\delta(A))$. The complement of δ-closed set is called δ-open. The δ-interior of set A in X and will denoted by $int_\delta(A)$ consists of those points x of A such that for some open set U containing x, $U\subseteq int(cl(U)) \subseteq A$. A point $x\in X$ is said to be a

θ-interior point of A [14] if there exists an open set U containing x such that

$U \subseteq cl(U) \subseteq A$. The set of all θ-interior points of A is said to be the θ-interior set and a subset A of X is called θ-open if $A = int_\theta(A)$.

Definition 1.1. A subset A of a space (X, τ) is called:

(1) α-open [13] if $A \subseteq int(cl(int(A)))$,

(2) preopen [12] if $A \subseteq int(cl(A))$,

(3) M-open [8] if $A \subseteq cl(int_\theta(A)) \cup int(cl_\delta(A))$,

(4) e-open [7] if $A \subseteq cl(int_\delta(A)) \cup int(cl_\delta(A))$.

The complement of α-open (resp. preopen, M-open, e-open) sets is called α-closed [13] (resp. pre-closed, M-closed, e-closed). The intersection of all α-closed (resp. pre-closed, M-closed, e-closed) sets containing A is called the α-closure (resp. pre-closure, M-closure, e-closure) of A and is denoted by α-cl(A) (resp. pcl(A), M-cl(A), e-cl(A)). The union of all α-open (resp. preopen, M-open, e-open) sets contained in A is called the α-interior (resp. pre-interior, M-interior, e-interior) of A and is denoted by α-int(A) (resp. pint(A), M-int(A), e-int(A)). The family of all M-open (resp. M-closed) sets in a space (X, τ) is denoted by $MO(X, \tau)$ (resp. $MC(X, \tau)$).

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Definition 1.2. A subset A of a space(X, τ) is called:

(1) generalized closed (= g-closed) set [2] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open,

(2) generalized α-closed (= gα-closed) set [3] if α-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open,

(3) generalized pre-closed (= gp-closed) set [11] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open,

(4) generalized e-closed (= ge-closed) set [4] if e-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

The complement of generalized e-closed (=ge-closed) set is called generalized e-open (= ge-open).

II. GENERALIZED M-CLOSED SETS

Definition 2.1. A subset B of a topological space (X,τ) is called a generalized M-closed (= gM-closed) set if M-cl(B) $\subseteq$ U whenever B $\subseteq$ U and U is open in (X, τ).

The family of all generalized M-closed sets of a space X is denoted by GMC(X).

Remark 2.1. for The following diagram holds for each a subset A of X.

$$\begin{array}{ccccccccc}
\text{closed} & \rightarrow & \alpha\text{-closed} & \rightarrow & \text{pre-closed} & \rightarrow & \text{M-closed} & \rightarrow & \text{e-closed} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{g-closed} & \rightarrow & \text{g}\alpha\text{-closed} & \rightarrow & \text{gp-closed} & \rightarrow & \text{gM-closed} & \rightarrow & \text{ge-closed}
\end{array}$$

None of these implications are reversible as shown in the following examples. Other examples are as shown in [2, 3, 4, 11].

Example 2.1. Let X = {a, b, c, d} and τ = {X, ϕ, {a}, {c}, {a, c}, {a, b, c}, {a, c, d}}. Then {a, b} of X is ge-closed but not gM-closed,

Example 2.2. Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}. Then {b} is an gM-closed set but not gp-closed.

Theorem 2.1. The arbitrary intersection of any gM-closed subsets of X is also a gM-closed subset of X.

Proof. Let $\{A_i : i \in I\}$ be any collection of gM-closed subsets of X such that $\bigcap_{i=1} A_i \subseteq H$ and H be open set in X. But, A_i is a gM-closed subset of X, for each $i \in I$, then M-cl(A_i) $\subseteq$ H, for each $i \in I$. Hence $\bigcap_{i=1}$ M-cl(A_i) $\subseteq$ H, for each $i \in I$. Therefore M-cl($\bigcap_{i=1} A_i$) $\subseteq$ H. So, $\bigcap_{i=1} A_i$ a gM-closed subset of X.

Remark 2.2. The union of two gM-closed subsets of X need not be a gM-closed subset of X.

Let be X = {a, b, c} with topology τ = {X, $\varnothing$, {a}, {b},{a, b}}. Then two subsets {a} and {b} of X are gM-closed subsets but their union {a, b} is not a gM-closed subset of X.

Theorem 2.2. A subset A of a space (X, τ) is gM-closed if and only if for each A $\subseteq$ H and H is M-open, there exists a M-closed set F such that A $\subseteq$ F $\subseteq$ H.

Proof. Suppose that A is a gM-closed subset of X, A $\subseteq$ H and H is a M-open set. Then M-cl(A) $\subseteq$ H. If we put F = M-cl(A) hence A $\subseteq$ F $\subseteq$ H.

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Conversely. Assume that $A \subseteq H$ and H is a M-open set. Then by hypothesis there exists a M-closed set F such that $A \subseteq F \subseteq H$. So, $A \subseteq$ M-cl(A) $\subseteq F$ and hence M-cl(A) $\subseteq H$. Therefore, A is gM-closed.

Lemma 2.1. Let A be θ-closed and B is M-closed, then $A \cup B$ is M-closed.

Theorem 2.3. If A is a θ-closed and B is a gM-closed subset of a space X, then $A \cup B$ is also gM-closed.

Proof. Suppose that $A \cup B \subseteq H$ and H is a M-open set. Then $A \subseteq H$ and $B \subseteq H$. But B is gM-closed, then M-cl(B) $\subseteq H$ and hence $A \cup B \subseteq A \cup$ M-cl(B) $\subseteq H$. But $A \cup$ M-cl(B) a M-closed set. Hence there exists a M-closed set $A \cup$ M-cl(B) such that $A \cup B \subseteq A \cup$ M-cl(B) $\subseteq H$. Thus by Theorem 2.2, $A \cup B$ is gM-closed.

Theorem 2.4. For an element $p \in X$, the set $X \setminus \{p\}$ is gM-closed or M-open.

Proof. Suppose that $X \setminus \{p\}$ is not a M-open set. Then X is the only M-open set containing $X / \{p\}$. This implies that M-cl($X \setminus \{p\}$) $\subseteq X$. Hence $X \setminus \{p\}$ is a gM-closed set in X.

Theorem 2.5. If A is a gM-closed set of X such that $A \subseteq B \subseteq$ M-cl(A),then B is a gM-closed set in X.

Proof. Let H be an open set of X such that $B \subseteq H$. Then $A \subseteq H$. But A is a gM-closed set of X, then M-cl(A) $\subseteq H$. Now, M-cl(B) $\subseteq$ M-cl(M-cl(A))= M-cl(A) $\subseteq H$. Therefore B is a gM-closed set in X.

Theorem 2.6. Let A be a gM-closed subset of (X, τ). Then M-cl(A)\A contains no nonempty closed set in X.

Proof. Let F be a closed subset of M-cl(A)\A. Since X\F is open, $A \subseteq$ X\F and A is gM-closed, it follows that M-cl(A) $\subseteq$ X\F and thus $F \subseteq$ X\M-cl(A). This implies that $F \subseteq$ (X\M-cl(A)) $\cap$ (M-cl(A) \ A) = $\varnothing$ and hence F = $\varnothing$.

Corollary 2.1. gM-closed subset A of a topological space X is M-closed if and only if M-cl(A)\A is closed.

Proof. Let A be gM-closed set. If A is M-closed, then we have M-cl(A)\A = $\varnothing$ which is closed set. Conversely, let M-cl(A)\A be closed. Then, by Theorem 2.6, M-cl(A)\A does not contain any non-empty closed subset and since M-cl(A)\A is closed subset of itself, then M-cl(A)\A = $\varnothing$. This implies that A = M-cl(A) and so A is M-closed set.

Theorem 2.7. If A is open and an gM-closed sets of X, then A is a gM-closed set in X.

Proof . Let H be any open set in X such that $A \subseteq H$. Since A is open and an gM-closed sets of X, then M-cl(A) $\subseteq$ A. Then M-cl(A) $\subseteq A \subseteq H$. Hence A is a gM-closed set in X.

Theorem 2.8. If A is both open and a gM-closed subsets of a topological space (X, τ), then A is M-closed.

Proof . Assume that A is both an open and a gM-closed subsets of a topological space (X, τ). Then M-cl(A) $\subseteq$ A. Hence A is M-closed.

Theorem 2.9. If A is both open and a gM-closed subsets of X and F is a θ-closed set in X, then $A \cap F$ is a gM-closed set in X.

Proof . Let A be an open and a gM-closed subsets of X and F be a θ-closed set in X. Then by Theorem 2.8, A is M-closed. So, $A \cap F$ is M-closed . Therefore, $A \cap F$ is a gM-closed set in X.

Theorem 2.10. [10] If A is a θ-open set and H is an M-open set in a topological space (X, τ), then $A \cap H$ is an M-open set in X.

Theorem 2.11. If A is both an open and a g-closed subsets of A, then A is a gM-closed set in X.

Proof. Let A be open and a g-closed subsets of X and $A \subseteq H$, where H is an open set in X. Then by hypothesis, M-cl(A) $\subseteq$ cl(A) $\subseteq A$, that is M-cl(A) $\subseteq H$. Thus A is a gM-closed set in X.

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Theorem 2.12. For a topological space (X, τ), then $MO(X, \tau) \subseteq \{F \subseteq X$: F is closed$\}$ if and only if every subset of X is a gM-closed subset of X.

Proof. Suppose that $MO(X, \tau) \subseteq \{F \subseteq X$: F is closed$\}$. Let A be any subet of X such that $A \subseteq H$, where H is an M-open set in X. Then $H \in MO(X, \tau) \subseteq \{F \subseteq X$: F is closed$\}$. That is $H \in \{F \subseteq X$: F is closed$\}$.Thus H is an M-closed set . Then M-cl(H) = H . Also, M-cl(A) $\subseteq$ M-cl(H) $\subseteq$ H . Hence A is a gM-closed subset of X.

Conversely. Suppose that every subset of (X, τ) is a gM-closed subset in X and $H \in MO(X, \tau)$. Since $H \subseteq H$ and H is gM-closed, we have M-cl(H) $\subseteq$ H. Thus M-cl(H) = H and $H \in \{F \subseteq X$: F is closed$\}$. Therefore $MO(X, \tau) \subseteq \{F \subseteq X$: F is closed$\}$.

Definition 2.2. The intersection of all M-open subsets of (X, τ) containing A is called the M-kernel of A and is denoted by M-ker(A).

Lemma 2.2. For any subset A of a toplogical space (X, τ), then A $\subseteq$ M-ker(A).

Proof. Follows directly from Definition 2.2.

Lemma 2.3. Let (X, τ) be a topological space and A be a subset of X. If A is M-open in X, then M-ker(A) = A.

Theorem 2.13. A subset A of a topological space X is gM-closed if and only if M-cl(A) $\subseteq$ M-ker(A).

Proof. Since A is gM-closed, M-cl(A) $\subseteq$ G for any open set G with $A \subseteq G$ and hence M-cl(A) $\subseteq$ M-ker(A).

Conversely, let G be any open set such that $A \subseteq G$. By hypothesis, M-cl(A) $\subseteq$ M-ker(A) $\subseteq$ G and hence A is gM-closed.

III. GENERALIZED M-OPEN SETS

Definiton 3.1. A subset A of a topological space (X, τ) is called a generalized M-open (breifly, gM-open) set in X if X\A is gM-closed in X. We denote the family of all gM-open sets in X by GMO(X).

Theorem 3.1. Let (X, τ) be a topological space and $A \subseteq X$. Then the following statements are equivalent:

(1) A is a gM-open set,

(2) for each closed set F contained in A, F $\subseteq$ M-int(A),

(3) for each closed set F contained in A, there exists an M-open set H such that $F \subseteq H \subseteq A$.

Proof. (1) $\rightarrow$(2). Let $F \subseteq A$ and F be a M-closed set. Then X\A $\subseteq$ X\F which is M-open of X , hence M-cl(X\A) $\subseteq$ X\F. So, F $\subseteq$ M-int(A).

(2) $\rightarrow$(3). Suppose that $F \subseteq A$ and F be an M-closed set. Then by hypothesis, F $\subseteq$ M-int(A). But H = M-int(A), hence there exists a M-open set H such that $F \subseteq H \subseteq A$.

(3) $\rightarrow$ (1). Assume that X\A $\subseteq$ V and V is a M-open set of X. Then by hypothesis, there exists an M-open set H such that X\V $\subseteq H \subseteq A$, that is, X\A $\subseteq$ X\H $\subseteq$ V. Therefore, by Theorem 2.2, X\A is gM-closed in X and hence A is gM-open in X.

Theorem 3.2. If A is θ-open and B is a gM-open subsets of a space X, then $A \cap B$ is also gM-open.

Proof. Follows from Theorem 2.3.

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Proposition 3.1. If M-int(A) $\subseteq$ B $\subseteq$ A and A is a gM-open set of X, then B is gM-open.

Proposition 3.2. Let A be M-closed and a gM-open sets of X. Then A is M-open.

Proof. Let A be a M-closed and a gM-open sets of X. Then A $\subseteq$ M-int(A) and hence A is M-open.

Theorem 3.3. For a space (X, τ), if A is a gM-closed set of X, then M-cl(A)\A is gM-open.

Proof . Suppose that A is a gM-closed set of X and F is a M-closed set contained in M-cl(A)\A. Then by Theorem 2.2 , F = $\varnothing$ and hence F $\subseteq$ M-int(M-cl(A)\A). Therefore, M-cl(A)\A is gM-open.

Theorem 3.4. If A is a gM-open subset of a space (X, τ), then G = X, whenever G is M-open and M-int(A) $\cup$ (X\A) $\subseteq$ G.

Proof. Let G be an open set of X and M-int(A) $\cup$ (X\A) $\subseteq$ G. Then X\G $\subseteq$ (X\M-int(A)) $\cap$ A = M-cl(X\A) \ (X\A). Since X\G is closed and X\A is gM-closed, by Theorem 2.6, X\G = $\varnothing$ and hence G = X.

Theorem 3.5. For a topological space (X, τ), then every singleton of X is eiher gM-open or M-open.

Proof. Let (X,τ) be a topological space and p$\in$X. To prove that $\{p\}$ is eiher gM-open or M-open. That is to prove $X \setminus \{p\}$ is eiher gM-closed or M-open, which follows directly from Theorem 2.4.

IV. M-$T_{1/2}$ SPACES AND GENERALIZED FUNCTIONS

Definition 4.1. A space (X, τ) is called an M-$T_{1/2}$ -space if every gM-closed set is M-closed.

Theorem 4.1. For a topological space (X, τ), the following conditions are equivalent:

(1) X is M-$T_{1/2}$

(2) Every singleton of X is either closed or M-open.

Proof. (1)$\to$ (2). Let $p \in X$ and assume that $\{p\}$ is not closed. Then $X \setminus \{p\}$ is not open and hence $X \setminus \{p\}$ is gM-closed. Hence, by hypothesis, $X \setminus \{p\}$ is M-closed and thus $\{p\}$ is M-open.

(2) $\to$ (1). Let A $\subseteq$ X be gM-closed and p $\in$M-cl(A). We will show that $p \in A$. For consider the following two cases:

Case (i). The set $\{p\}$ is closed. Then, if $p \notin A$, then there exists a closed set in M-cl(A)\A. Hence by Corollary 2.1, $p \in A$.

Case (ii). The set $\{p\}$ is M-open. Since p $\in$M-cl(A), then $\{p\}$ $\cap$ M-cl(A) $\neq \varnothing$ .Thus $p \in A$. So, in both cases, $p \in A$. This shows that M-cl(A) $\subseteq$ A or equivalently, A is M-closed.

Theorem 4.2. For a topological space (X, τ), the following hold:

(1) MO(X, τ) $\subseteq$ GMO(X, τ),

(2) The space X is M-$T_{1/2}$ if and only if MO(X, τ) = GMO(X, τ).

Proof. (1) Let A be a M-open set. Then X\A is M-closed and so gM-closed . This implies that A is gM-open. Hence MO(X, τ) $\subseteq$ GMO(X, τ) .

(2) $\Rightarrow$: Let (X, τ) be a M-$T_{1/2}$ and $A \in GMO(X, \tau)$. Then $X \backslash A$ is gM-closed. By hypothesis, $X \backslash A$ is M-closed and thus A is M-open this implies that $A \in MO(X, \tau)$. Hence, $MO(X, \tau) = GMO(X, \tau)$.

$\Leftarrow$: Let be $MO(X, \tau) = GMO(X, \tau)$ and A a gM-closed set. Then $X \backslash A$ is gM-open. Hence $X \backslash A \in MO(X, \tau)$. Thus A is M-closed. Therefore (X, τ) is M-$T_{1/2}$.

Definition 4.2. A function $f : X \to Y$ is called :

(1) gM-continuous if $f^{-1}(F)$ is gM-closed in X, for every closed set F of Y,

(2) M-gM-continuous if $f^{-1}(F)$ is gM-closed in X, for every M-closed set F of Y,

(3) gM-irresolute if $f^{-1}(F)$ is gM-closed in X for every gM-closed set F of Y.

(4) e-ge-continuous if $f^{-1}(F)$ is ge-closed in X, for every e-closed set F of Y,

Definition 4.3. A function $f : X \to Y$ is called :

(1) ge-continuous [4] if $f^{-1}(F)$ is ge-closed in X, for every closed set F of Y,

(2) ge-irresolute [4] if $f^{-1}(F)$ is ge-closed in X for every ge-closed set F of Y,

(3) M-continuous [10] if $f^{-1}(F)$ is M-closed in X, for every closed set F of Y,

(4) M-irresolute [10] if $f^{-1}(F)$ is M-closed in X for every M-closed set F of Y.

Remark 4.1. The following diagram holds for a function $f : (X, \tau) \to (Y, \sigma)$:

eg-irresoluteness $\to$ e-ge-continuity $\to$ ge-continuity

$\uparrow$ $\uparrow$

gM-irresoluteness $\to$ M-gM-continuity $\to$ gM-continuity

$\uparrow$ $\uparrow$

M-irresoluteness $\to$ *M-continuity*

The converses of the above implications are not true in general as is shown by [4, 10] and the following example.

Example 4.1. In Example 2.1, Let $f: (X, \tau) \to (X, \tau)$ be a function defined as follows: $f(a) = a$, $f(b) = b$, $f(c)= c$, $f(d) = e$ and $f(e) = d$. Then f is ege-continuous (resp. ge-continuous) but it is not MgM-continuous (resp. gM-continuous).

Example 4.2. Let $X = Y=\{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma =\{Y, \varnothing, \{b, c\}\}$. Then a mapping $f : (X, \tau)\to (Y, \sigma)$ which defined by the identity mapping is gM-continuous but it is neither M-gM-continuous nor

gM-irresolute.

Example 4.3. Let $X = Y=\{a, b, c, d\}$ with $\tau = \{X, \varnothing, \{a\}, \{c\}, \{a,c\},\{a, b\}, \{a, b, c\},\{a, c, d\}\}$ and $\sigma =\{Y, \varnothing, \{b\}, \{a, b\},\{b, d\},\{a, b, d\}\}$. Then a mapping

$f : (X, \tau) \to (Y, \sigma)$ which defined by the identity mapping is gM-continuous but it is neither M-continuous nor M-irresolute.

Example 4.4. Let $X = Y=\{a, b, c, d\}$ with $\tau = \{X, \varnothing, \{a\}, \{c\}, \{a,c\}, \{a, b, c\}, \{a, c, d\}\}$ and $\sigma =\{Y, \varnothing, \{a, b\},\{c, d\}\}$. Then a mapping $f : (X, \tau) \to (Y, \sigma)$ which defined by the identity mapping is ge-continuous but it is not gM-continuous.

Theorem 4.3. Let $f : X \to Y$ and $h : Y \to Z$ be functions.

(1) If f is gM-irresolute and h is gM-continuous, then the composition

$h \circ f: X \to Z$ is gM-continuous.

(2) If f is gM-continuous and h is continuous, then the composition $h \circ f: X \to Z$ is gM-continuous.

(3) If f and h are gM-irresolute, then the composition $h \circ f : X \to Z$ is gM-irresolute.

(4) If f is gM-irresolute and h is M-gM-continuous, then the composition

$h \circ f: X \to Z$ is

M-gM-continuous.

(5) If f and h are M-gM-continuous and Y is M-$T_{1/2}$, then the composition

$h \circ f : X \to Z$ is M-gM-continuous.

Theorem 4.4. If a function $f : X \to Y$ is M-gM-continuous and Y is M-$T_{1/2}$, then f is gM-irresolute.

Proof. Let F be any gM-closed subset of Y. Since Y is M-$T_{1/2}$, then F is M-closed in Y. Hence $f^{-1}(F)$ is gM-closed in X. This show that f is gM-irresolute.

Theorem 4.5. If a function $f: X \to Y$ is gM-continuous and X is M-$T_{1/2}$, then f is M-continuous.

Proof. Let F be any closed set of Y. Since f is gM-continuous, $f^{-1}(F)$ is gM-closed in X and then $f^{-1}(F)$ is M-closed in X. Hence f is M-continuous.

Theorem 4.6. If a function $f: X \to Y$ is M-gM-continuous and X is M-$T_{1/2}$, then f is M-irresolute.

Proof. Let F be any M-closed set of Y. Since f is M-gM-continuous, $f^{-1}(F)$ is gM-closed in X and then $f^{-1}(F)$ is M-closed in X. Hence f is M-irresolute.

Definition 4.4. A function $f : X \to Y$ is said to be:

(1) gM-closed if f(A) is gM-closed in Y, for each closed set A of X.

(2) M-gM-closed if f(A) is gM-closed in Y, for each M-closed set A of X.

Theorem 4.7. If $f : X \to Y$ is closed M-gM-continuous, then $f^{-1}(K)$ is gM-closed in X for each gM-closed set K of Y.

Proof. Let K be a gM-closed set of Y and U an open set of X containing $f^{-1}(K)$. Put $V = Y\backslash f(X\backslash U)$, then V is open in Y, $K \subseteq V$, and $f^{-1}(V) \subseteq U$. Therefore, we have $M\text{-}cl(K) \subseteq V$ and hence $f^{-1}(K) \subseteq f^{-1}(M\text{-}cl(K)) \subseteq f^{-1}(V) \subseteq U$. Since f is M-gM-continuous, $f^{-1}(M\text{-}cl(K))$ is gM-closed in X and hence $M\text{-}cl(f^{-1}(K)) \subseteq M\text{-}cl(f^{-1}(M\text{-}cl(K))) \subseteq U$. This shows that $f^{-1}(K)$ is gM-closed in X.

Corollary 4.1. If $f : X \to Y$ is closed M-irresolute, then $f^{-1}(K)$ is gM-closed in X for each gM-closed set K of Y.

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Theorem 4.8. If $f : X \to Y$ is an open M-gM-continuous bijection, then $f^{-1}(K)$ is gM-closed in X, for every gM-closed set K of Y.

Proof. Let K be a gM-closed set of Y and U an open set of X containing $f^{-1}(K)$. Since f is an open surjective, $K = f(f^{-1}(K)) \subseteq f(U)$ and f(U) is open. Therefore, $M\text{-}cl(K) \subseteq f(U)$. Since f is injective, $f^{-1}(K) \subseteq f^{-1}(M\text{-}cl(K)) \subseteq f^{-1}(f(U)) = U$. Since f is M-gM-continuous, $f^{-1}(M\text{-}cl(K))$ is gM-closed in X and hence

$M\text{-}cl(f^{-1}(K)) \subseteq M\text{-}cl(f^{-1}(M\text{-}cl(K))) \subseteq U$. therefore $f^{-1}(K)$ is gM-closed in X.

Definition 4.5. [9] A space X is said to be M-normal if for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint M-open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Theorem 4.9. Let $f : X \to Y$ be a closed M-gM-continuous injection. If Y is M-normal, then X is M-normal.

Proof. Let F_1 and F_2 be disjoint closed sets of X. Since f is a closed injection, then $f(F_1)$ and $f(F_2)$ are disjoint closed sets of Y. By the M-normality of Y, there exist disjoint $V_1, V_2 \in MO(Y)$ such that $f(F_i) \subseteq V_i$, for i = 1, 2. Since f is M-gM-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint gM-open sets of X and $F_i \subseteq$

$f^{-1}(V_i)$ for i = 1,2. Now, by Theorem 3.1, there exists an M-open sets U_i such that $F_i \subseteq U_i \subseteq f^{-1}(V_i)$ for i = 1,2. Then, $U_i \in MO(X, \tau)$, $F_i \subseteq U_i$ and

$U_1 \cap U_2 = \varnothing$. Therefore X is M-normal.

Corollary 4.2. If f: $X \to Y$ is a closed M-irresolute injection and Y is M-normal, then X is M-normal.

Proof. This is an immediate consequence since every M-irresolute function is M-gM-continuous.

Lemma 4.1. A surjection $f : X \to Y$ is M-gM-closed if and only if for each subset B of Y and each M-open set U of X containing $f^{-1}(B)$, there exists a gM-open set of V of Y such that

$B \subseteq V$ and $f^{-1}(V) \subseteq U$.

Theorem 4.10. If $f : X \to Y$ is a M-gM-closed continuous surjection and X is M-normal, then Y is M-normal.

Proof. Let F_1 and F_2 be any disjoint closed sets of Y. Then $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint closed sets of X. Since X is M-normal, there exist disjoint M-open sets U and V such that $f^{-1}(F_1) \subseteq U$ and $f^{-1}(F_2) \subseteq V$. By Lemma 4.1, there exist gM-open sets G and H of Y such that $F_1 \subseteq G$, $F_2 \subseteq H$, $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$. Since U and V are disjoint, G and H are disjoint. By Theorem 3.1, we have $F_1 \subseteq M\text{-}int(G)$, $F_2 \subseteq M\text{-}int(H)$ and $M\text{-}int(G) \cap M\text{-}int(H) = \varnothing$. Therefore, Y is M-normal.

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